Normal sampling and Modeling 8.3

Calculation of normal distributions using z-score

Normal approximation to the binomial distribution.

Normal distribution is a continuous distribution of the data. If the probability is **binomial** ie (discrete), and you have enough data, you can use normal distribution as a good approximation.

This approximation might be easier than using BINOMIAL formulas many times. Recall that

$P(x = a) = {}_nC_a(p)^a(q)^{n-a}$

Recall that z score: $z = \dfrac{x - \mu}{\sigma}$ also, for binomial data

$E(x) = \mu = np$ and $\sigma = \sqrt{npq}$

Example 1

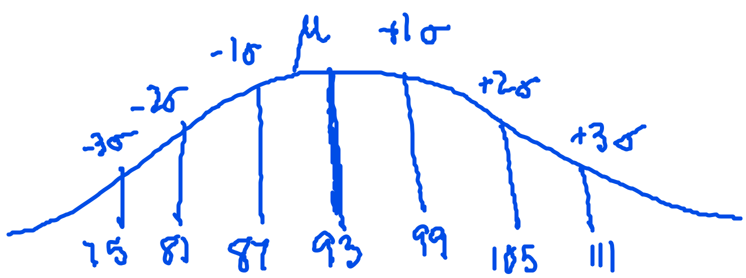
The mean number of points scored in an NBA game is 93 with a standard deviation of 6. Find the probability that a randomly selected game will see a score of…

a) Less than 83 points

b) More than 107 points

c) Between 87 and 99 points

d) Exactly 95 points



so, what we did in 8.2 won't work since 83 points is not exactly a # of σ's above or below μ.

z-scores come into play!

. take our actual #'s and convert to z-scores.

(a) $P(X < 83) = P(Z < -1.67)$

$= .0475$

or 4.75%

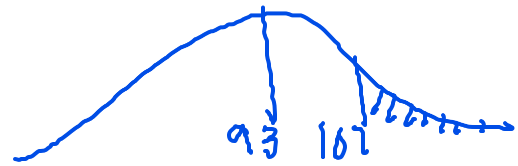
∴ need shaded

$z = \frac{x-\mu}{\sigma}$

$= \frac{83-93}{6}$

$= -1.67$

"area under the curve"



(b) $P(X > 107)$

$= 1 - P(X < 107)$

$= 1 - P\left(Z < \frac{107-93}{6}\right)$

$= 1 - P(Z < 2.33)$

$= 1 - .9901$

$= .0099$ or about 1%

but table uses shaded area less than # $\therefore$ 1 − P(white)

(c) $P(87 < X < 99)$

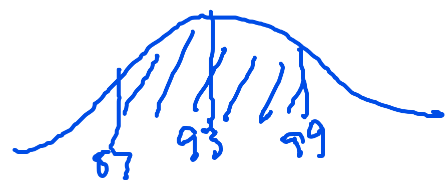
$= P(X < 99) - P(X < 87)$

$= P\left(Z < \frac{99-93}{6}\right) - P\left(Z < \frac{87-93}{6}\right)$

$= P(Z < 1) - P(Z < -1) \rightarrow$ * Note: between 1 st. dev. either side of μ $\therefore$ should be 68%

$= .8413 - .1587$

$= .6826$ or 68%



so all less than 99 subtract less than 87 (white)

(d) $P(X = 95)$

$= P(94.5 < X < 95.5)$

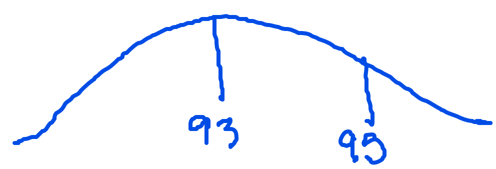
$= P(X < 95.5) - P(X < 94.5)$

$= P\left(Z < \frac{95.5-93}{6}\right) - P\left(Z < \frac{94.5-93}{6}\right)$

$= P(Z < .42) - P(Z < .25)$

$= .6629 - .5987$

$= .0642$ or 6%



take just before & just after.

Ex1 . A bank found that 24% of loans are delinquent. If 200 people with loans are selected at random, what is the probability that at least 60 are delinquent?

$P(X \geq 60)$ but need μ, σ $\rightarrow$ $p = .24$, $q = .76$

$\mu = np = 200(.24) = 48$

$\sigma = \sqrt{npq} = \sqrt{200(.24)(.76)} = 6.0399$

$= 1 - P(X < 60)$

$= 1 - P\left(Z < \frac{60-48}{6.0399}\right)$

$= 1 - P(Z < 1.99)$

$= 1 - .9767$

$= .0233$ or 2%

3. There are 10000 open heart surgeries at the three main downtown hospitals in Toronto per year

The probability for each success 0.96.

Find the probability of there being 9650 successes

$p = .96$

$q = .04$

$\mu = np$
$= 10000(.96)$
$= 9600$

$\sigma = \sqrt{npq}$
$= \sqrt{10\,000(.96)(.04)}$
$= 19.5959$

$P(X = 9650)$
$= P(9649.5 < X < 9650.5)$
$= P(X < 9650.5) - P(X < 9649.5)$
$= P\left(Z < \frac{9605.5 - 9600}{19.5959}\right) - P\left(Z < \frac{9649.5 - 9600}{19.5959}\right)$
$\approx P(Z < 2.58) - P(Z < 2.52)$
$= .9951 - .9941$
$= .0010$ or $.1\%$ (for exactly 9650 successes)

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